Existential Abstraction on Argumentation Frameworks via Clustering Supplement with Proof Details

Zeynep G. Saribatur[1] , **Johannes P. Wallner**[2]
[1]Institute of Logic and Computation, TU Wien, Austria
[2]Institute of Software Technology, Graz University of Technology, Austria
zeynep@kr.tuwien.ac.at, wallner@ist.tugraz.at

Proof Details

We give proofs of the formal statements in the paper.

Proof of Theorem 1. Let $F = (A, R)$ be an AF, m a mapping on A, and $\hat{F} = m(F)$ be the clustered AF according to m. Let $E \in \sigma(F)$. It remains to show that $\hat{E} = m(E) \in \hat{\sigma}(\hat{F})$. For $\sigma = cf$, suppose $\hat{E} \notin \hat{cf}(\hat{F})$. By definition, there are $\hat{a}$ and $\hat{b}$ in $single(\hat{A}) \cap \hat{E}$ s.t. $(\hat{a}, \hat{b}) \in \hat{R}$. This implies that there are a and b in E with $m(a) = \hat{a}$ and $m(b) = \hat{b}$ and $(a, b) \in R$ (since both $\hat{a}$ and $\hat{b}$ are singleton clusters). This contradicts E being conflict-free. We conclude that $\hat{E} \in \hat{cf}(\hat{F})$. For $\sigma = adm$, similarly as before suppose that $\hat{E} \notin \hat{adm}(\hat{F})$. By definition, either (i) $\hat{E} \notin \hat{cf}(\hat{F})$, or (ii) there is an $\hat{a} \in single(\hat{A}) \cap \hat{E}$ with $(\hat{b}, \hat{a}) \in \hat{R}$ s.t. there is no $\hat{c} \in \hat{E}$ with $(\hat{c}, \hat{b}) \in \hat{R}$. If $\hat{E} \notin \hat{cf}(\hat{F})$, we arrive at a contradiction as above (E would not be conflict-free). Suppose (ii) holds. Then there is an $a \in E$ with $m(a) = \hat{a}$. Since $|\hat{a}| = 1$, it follows that there is a $b \in \hat{b}$ s.t. $(b, a) \in R$ (one argument in the cluster must attack a). Since, by our supposition, there is no $\hat{c} \in \hat{E}$ that attacks $\hat{b}$ in $\hat{F}$, it follows that there is no $c \in E$ with $(c, b) \in R$ (otherwise such a c would be part of some cluster in $\hat{E}$ and the attack, as well). This contradicts $E \notin adm(F)$. We infer that $\hat{E} \in \hat{adm}(\hat{F})$. For $\sigma = stb$, similarly as before suppose that $\hat{E} \notin \hat{stb}(\hat{F})$. As above, we can conclude that $\hat{E} \in \hat{cf}(\hat{F})$. We show that $\hat{E}$ satisfies the two remaining conditions of $\hat{stb}$ extensions, by supposing that they do not hold and deriving a contradiction. Suppose there is a $\hat{b} \notin \hat{E}$ and there is no $\hat{a} \in \hat{E}$ with $(\hat{a}, \hat{b}) \in \hat{R}$. We directly arrive at a contradiction: there must be a $b \in \hat{b}$ s.t. $b \notin E$ and there is no $a \in E$ with $(a, b) \in R$ (otherwise there would be an $\hat{a} \in \hat{E}$ with $(\hat{a}, \hat{b}) \in \hat{R}$). Finally, assume that $\hat{E}$ does not attack an $\hat{a} \in \hat{E}$. Suppose that there is a $\hat{b} \in single(\hat{E})$ and $(\hat{a}, \hat{b}) \in \hat{R}$ (which would contradict $\hat{E}$ being stable, third condition of $\hat{stb}$). If there is no clustered argument in $\hat{E}$ that attacks $\hat{a}$ in $\hat{F}$ then no argument in E attacks any argument in $\hat{a}$ in F. Since E is stable in F, this implies that $\hat{a} \subseteq E$ (all arguments in $\hat{a}$ are not attacked by E and must then be part of E). Since $(\hat{a}, \hat{b}) \in \hat{R}$ and $|\hat{b}| = 1$, we infer that one argument $a \in \hat{a}$ attacks $\hat{b}$ in F (one argument in $\hat{a}$ must attack $\hat{b}$ since $\hat{b}$ is a singleton). Since the $\hat{b}$ is a singleton and all of $\hat{a}$ are in E we infer that E attacks $\hat{b}$ in F. But then E is not conflict-free, since $\hat{b} \in \hat{E}$ implies $\hat{b} \in E$, a contradiction. We infer that $\hat{E} \in \hat{stb}(\hat{F})$. □

Proof of Theorem 2. Let $\sigma = cf$. Suppose that there is an $\hat{E} \in \hat{cf}(\hat{F})$ and that $\hat{E} \notin \hat{\tau}(\hat{F})$. We show that there is an AF F with $F \in m^{-1}(\hat{F})$ and an $E \in cf(F)$ with $m(E) = \hat{E}$ (contradicting the claim that $\hat{\tau}$ abstracts conflict-free sets). Construct an AF $F = (A, R)$ as follows. Let A be the domain of m. Define $R' = \{(x, y) \mid (\hat{x}, \hat{y}) \in \hat{R}, x \in m^{-1}(\hat{x}), y \in m^{-1}(\hat{y})\}$ (i.e., R' contains an attack between x and y iff the corresponding clusters $\hat{x}$ and $\hat{y}$ attack in $\hat{R}$). For each $\hat{a} \in \hat{A}$ choose one $c_a \in \hat{a}$ (recall that we assume finite sets), and let $E = \{c_a \mid \hat{a} \in \hat{E}\}$. Define $\overline{R} = \{(c_x, c_y) \mid c_x, c_y \in E\}$. Set $R = R' \setminus \overline{R}$ (i.e., we remove from R' all attacks between arguments in E). We show that $m(F) = \hat{F}$. From construction we immediately get $m(A) = \hat{A}$. For $(x, y) \in R$, we infer that $(m(x), m(y)) \in \hat{R}$: if $(x, y) \in R$ then $(x, y) \in R'$, implying that $(m(x), m(y)) \in \hat{R}$. Thus, $m(R) \subseteq \hat{R}$. Let $(\hat{x}, \hat{y}) \in \hat{R}$. Consider first the case that $\hat{x} \neq \hat{y}$.

- If at most one of $\hat{x}$ or $\hat{y}$ is in $\hat{E}$, it follows that for all $x \in \hat{x}$ and $y \in \hat{y}$ we have $(x, y) \in R$ (this attack is in R' and not in $\overline{R}$).
- Consider the case that both of $\hat{x}$ or $\hat{y}$ are in $\hat{E}$. If $\{\hat{x}, \hat{y}\} \subseteq single(\hat{A})$, then $(\hat{x}, \hat{y}) \notin \hat{R}$ (since $\hat{E} \in \hat{cf}(\hat{F})$). Consider the case that one of $\hat{x}$ or $\hat{y}$ is a non-singleton cluster in $\hat{F}$, say $|\hat{x}| > 1$. Then there is an $x \in \hat{x}$ with $x \neq c_x$. Let $y \in \hat{y}$. It follows that $(x, y) \notin \overline{R}$, and, thus, $(x, y) \in R$.

If $\hat{x} = \hat{y}$ (self-attack $(\hat{x}, \hat{x}) \in \hat{R}$), then one can reason analogously: if $\hat{x} \notin \hat{E}$, then for all $x \in \hat{X}$ we have $(x, x) \in R$. If $\hat{x} \in \hat{E}$, then $\hat{x}$ is not a singleton. Again there is some $x \in \hat{x}$ with $x \neq c_x$, and $(x, x) \in R$. We conclude that $m(R) = \hat{R}$, and, in turn, $m(F) = \hat{F}$.

It remains to show that $E \in cf(F)$. Let $x, y \in E$. By construction of $\overline{R}$ it holds that $(x, y) \in \overline{R}$, and, thus, $(x, y) \notin R$. Finally for conflict-free sets, $m(E) = \hat{E}$, by construction.

Let $\sigma = adm$. Assume that $\hat{E} \in \hat{adm}(\hat{F})$. Suppose that $\hat{E} \notin \hat{\tau}(\hat{F})$. Construct an AF $F = (A, R)$ in a similar fashion as before. Let R' be as in the proof for *cf*, and choose arguments c_a for $\hat{a} \in \hat{A}$ as before. Again, let $E = \{c_a \mid \hat{a} \in \hat{E}\}$. Define $\overline{R_1} = \{(x, c_y) \mid x \in A, |\hat{y}| > 1, c_y \in E\}$ (attacks onto members of E whose cluster is non-singleton) and $\overline{R_2} = \{(c_x, c_y) \mid c_x, c_y \in E\}$ (attacks inside E). Let $R = R' \setminus (\overline{R_1} \cup \overline{R_2})$. We again get immediately that $m(A) = \hat{A}$. Let $(x, y) \in R$. Since $(x, y) \in R'$ we infer that $(m(x), m(y)) \in \hat{R}$, implying $m(R) \subseteq \hat{R}$. Let $(\hat{x}, \hat{y}) \in \hat{R}$. If $\hat{y} \notin \hat{E}$, it follows that for all $x \in \hat{x}$ and $y \in \hat{y}$ we have $(x, y) \in R$. Let $\hat{y} \in \hat{E}$. Consider the following subcases.

- $\hat{x} \in \hat{E}$. Consider again subcases depending which is a singleton cluster.
  - $|\hat{x}| = 1$ and $|\hat{y}| = 1$: if both clusters $\hat{x}$ and $\hat{y}$ are singletons in $\hat{F}$, then $(\hat{x}, \hat{y}) \notin \hat{R}$ (since $\hat{E} \in \hat{cf}(\hat{F})$).
  - $|\hat{x}| > 1$ and $|\hat{y}| = 1$: then for an $x \in \hat{x}$ with $x \neq c_x$, and $m(y) = \hat{y}$, we have $(x, y) \in R$ ((x, y) is not in $\overline{R_1}$ or $\overline{R_2}$).
  - $|\hat{x}| = 1$ and $|\hat{y}| > 1$: then for a $y \in \hat{y}$ with $y \neq c_y$, and $m(x) = \hat{x}$, we have $(x, y) \in R$ ((x, y) is not in $\overline{R_1}$ or $\overline{R_2}$).
  - $|\hat{x}| > 1$ and $|\hat{y}| > 1$: then for $x \in \hat{x}$ and $y \in \hat{y}$ we have $(x, y) \in R$ with $x \neq c_x$ and $y \neq c_y$.
- $\hat{x} \notin \hat{E}$. Consider again subcases depending which is a singleton cluster.
  - $|\hat{x}| = 1$ and $|\hat{y}| = 1$: then for x and y with $m(x) = \hat{x}$ and $m(y) = \hat{y}$ we have $(x, y) \notin \overline{R_1} \cup \overline{R_2}$.
  - $|\hat{x}| > 1$ and $|\hat{y}| = 1$: similar as the previous case (just take an arbitrary $x \in \hat{x}$).
  - $|\hat{x}| = 1$ and $|\hat{y}| > 1$: let $y \in \hat{y}$ with $y \neq c_y$ and x s.t. $m(x) = \hat{x}$. It follows that $(x, y) \notin \overline{R_1} \cup \overline{R_2}$ (we choose a different $y \in \hat{y}$ than the one removed by $\overline{R_1}$).
  - $|\hat{x}| > 1$ and $|\hat{y}| > 1$: similar as the previous case.

Thus, $\hat{R} \subseteq m(R)$, implying $m(R) = \hat{R}$. It remains to show that $E \in adm(F)$. It follows that $E \in cf(F)$ (similar arguments as above). Suppose that E does not defend itself. Then there is an $a \in E$ with some $(b, a) \in R$ such that there is no $c \in E$ with $(c, b) \in R$. If, for $m(a) = \hat{a}$, we have $|\hat{a}| > 1$, then $(b, a) \notin R$ (since $(b, a) \in \overline{R_1}$). If $|\hat{a}| = 1$, then there is a $\hat{b}$ with $m(b) = \hat{b}$ such that $(\hat{b}, \hat{a}) \in \hat{R}$. This implies (by $\hat{a} \in \hat{E}$ and $\hat{E} \in \hat{adm}(\hat{F})$) that there is a $\hat{c} \in \hat{E}$ such that $(\hat{c}, \hat{b}) \in \hat{R}$. By construction, $c_c \in E$. Since $(c_c, b) \notin \overline{R_1} \cup \overline{R_2}$ (by $b \notin E$), it follows that $(c_c, b) \in R$, contradicting that $E \notin adm(F)$. It follows that $E \in adm(E)$, and $m(E) = \hat{E} \in \hat{\tau}(\hat{F})$.

Let $\sigma = stb$. Assume that $\hat{E} \in \hat{stb}(\hat{F})$. Suppose that $\hat{E} \notin \hat{\tau}(\hat{F})$. Construct an AF $F = (A, R)$ in a similar fashion as before. Let R' be as in the proof for *cf*, and choose arguments c_a for $\hat{a} \in \hat{A}$ as before. Let $E = \{c_a \mid \hat{a} \in \hat{E}\} \cup \{x \mid x \in \hat{x}, \hat{x} \in \hat{E} \text{ s.t. } \nexists(\hat{y}, \hat{x}) \in \hat{R} \text{ with } \hat{y} \in \hat{E}\}$ (include here also full clusters that are unattacked or only attacked from outside $\hat{E}$). Define $\overline{R} = \{(x, y) \mid x, y \in E\}$. Set $R = R' \setminus \overline{R}$ (i.e., similar as in the proof for *cf*, we remove from R' all attacks between arguments in E; however note that E is different). We now prove that $m(F) = \hat{F}$ holds. We infer that $m(A) = \hat{A}$, by construction. First, similarly as above, assume that $(x, y) \in R$. We have $(x, y) \in R'$. Then $(\hat{x}, \hat{y}) \in \hat{R}$ for $m(x) = \hat{x}$ and $m(y) = \hat{y}$. This implies that $m(R) \subseteq \hat{R}$. The other direction requires again a case analysis. Let $(\hat{x}, \hat{y}) \in \hat{R}$. Assume that either $\hat{x}$ or $\hat{y}$ is not in $\hat{E}$. Say $\hat{x} \notin \hat{E}$ (other case analogous). Then $(x, y) \in R$ for all $x \in \hat{x}$ and all $y \in \hat{y}$ (these attacks are in R' and not in $\overline{R}$, since the latter only contains arguments outside E, implying that their corresponding cluster is outside $\hat{E}$). Assume that $\{\hat{x}, \hat{y}\} \subseteq \hat{E}$. If $\hat{x} = \hat{y}$ then consider the following two cases.

- If $|\hat{x}| = 1$, then there is no $(\hat{x}, \hat{x}) \in \hat{R}$ due to conflict-freeness: if $\hat{E}$ is $\hat{cf}$ in $\hat{F}$ then there are no attacks between singletons within $\hat{E}$ in $\hat{F}$.
- If $|\hat{x}| > 1$, then $\hat{E}$ attacks $\hat{x}$, which means that $E \cap \hat{x} = \{c_x\}$ (only the "chosen" argument is in E, not all, since the clustered argument is attacked from $\hat{E}$). But then there is an $x \in \hat{x}$ with $x \neq c_x$ s.t. $(x, x) \in R$.

Consider the case that $\hat{x} \neq \hat{y}$.

- If $|\hat{x}| = |\hat{y}| = 1$ (both are singletons), then since both are in $\hat{E}$ there is no attack between these two clustered arguments (would contradict $\hat{E}$ being $\hat{stb}$).
- If $|\hat{y}| = 1$ and $|\hat{x}| > 1$, then, by definition of $\hat{stb}$ we infer that $\hat{E}$ attacks $\hat{x}$ in $\hat{F}$ (otherwise the third condition of $\hat{stb}$ would be violated). This implies that $E \cap \hat{x} = \{c_x\}$. This means that there is an $x \in \hat{x}$ with $x \neq c_x$ and $(x, c_y) \in R$.
- If $|\hat{y}| > 1$, then there is an $y \in \hat{y}$ s.t. $y \neq c_y$ (note that $\hat{E}$ attacks $\hat{y}$ in $\hat{F}$). Then $(c_x, y) \in R$. This case covers both subcases with $|\hat{x}| = 1$ and $|\hat{x}| > 1$.

It remains to show that $E \in stb(F)$. First, $E \in cf(F)$: if x and y in E, then there is no $(x, y) \in R$, since attacks between members of E are removed via $\overline{R}$. Let $b \in A$ and $b \notin E$. Consider two cases for the corresponding cluster $m(b) = \hat{b}$: (i) $\hat{b} \in \hat{E}$ and (ii) $\hat{b} \notin \hat{E}$. In case (i), then $\hat{b}$ must be attacked by an $\hat{a} \in \hat{E}$ (if unattacked or not attacked from $\hat{E}$ then all of $\hat{b}$ are in E). Then $(c_a, b) \in R$ (note that $b \neq c_b$). In case (ii), there must be an $\hat{a} \in \hat{E}$ s.t. $(\hat{a}, \hat{b}) \in \hat{R}$. Then $(a, b) \in R$ for $a \in \hat{a}$ (including c_a). Thus, $E \in stb(F)$, and $m(E) = \hat{E} \in \hat{\tau}(\hat{F})$. □

Proof of Proposition 3. For the first item, let $a \in E$ with $E \in \sigma(F)$. Then $\hat{F}' = (\{A\}, \hat{R}')$ under $\hat{\sigma}(\hat{F}') = \{\emptyset, \{A\}\}$ is faithful w.r.t. F under σ, if $m'(E) = A$ (if $A = \{a\}$ them a is unattacked in F; otherwise there are at least two arguments since E is conflict-free, which implies that clustered argument $\{A\}$ is non-singleton). For the second item, let $\hat{a}$ be a non-singleton cluster. It holds that $\{\hat{a}\} \in \hat{\sigma}(\hat{F})$. Due to faithfulness, we infer that there is an $E \in \sigma(F)$

with $m(E) = \{\hat{a}\}$. For the last item, if $X \in \hat{A}$, then $\{X\} \in \hat{\sigma}(\hat{F})$. This contradicts faithfulness. □

Proof of Corollary 4. Consider the negation of both statements. The following holds (since an abstracting $\hat{\sigma}'$ may not include $\hat{E}$ if there is no corresponding AF including E). Let $\hat{E} \subseteq \hat{E}$.

$$\forall F = (A, R) \text{ s.t. } m(F) = \hat{F} \text{ and}$$
$$\forall E \subseteq A \text{ s.t. } m(E) = \hat{E} : E \notin \sigma(F)$$
$$\text{iff } \exists \hat{\sigma}' \text{ abstracting } \sigma \text{ s.t. } \hat{E} \notin \hat{\sigma}'(\hat{F})$$
$$\text{iff } \hat{E} \notin \hat{\sigma}(\hat{F})$$

As an example, $\hat{\sigma}'$ can be defined as $\bigcup_{F \in m^{-1}(\hat{F})} m(\sigma(F))$ (exactly collecting all mapped σ-extensions for each F that maps to $\hat{F}$). Since $\hat{\sigma}(\hat{F})$ is included in any $\hat{\sigma}'(\hat{F})$ if $\hat{\sigma}'$ abstracts σ (Theorem 2), we can infer the statement of the corollary. □

Proof of Proposition 5. Consider the complementary problem: given a clustered AF $\hat{F}$ according to m, a $\hat{adm}$-extension $\hat{E}$, and an AF F with $m(F) = \hat{F}$, verify that there exists an $E \in \mathit{adm}(E)$ s.t. $m(E) = \hat{E}$. For membership in NP, consider a guess of E and checking the conditions. For hardness, we reduce from the problem of checking satisfiability of a Boolean formula $\phi = c_1 \wedge \cdots \wedge c_n$ with clauses c_i over vocabulary X. We use $\overline{x}$ to denote a negated literal in a clause. We utilize a variant of the standard construction for showing hardness for credulous reasoning on AFs under admissibility. Let $F = (A, R)$ be given by $A = \{x, \overline{x} \mid x \in X\} \cup \{c \in C\} \cup \{q\}$ and $R = \{(x, \overline{x}), (\overline{x}, x) \mid x \in X\} \cup \{(z, c) \mid z \in X \cup \overline{X}, z \in c, c \in C\} \cup \{(c, q) \mid c \in C\}$. Further, let $m(x) = m(\overline{x}) = \hat{x}$ for each $x \in X$, and each $c \in C$ and q mapped to themselves (i.e., we cluster $\{x, \overline{x}\}$). Finally, set $\hat{E} = \{q\} \cup \{x, \overline{x} \mid x \in X\}$. We claim that ϕ is satisfiable iff $\hat{E}$ is not spurious under admissibility. By previous results, it holds that there is an admissible set $E \in \mathit{adm}(F)$ containing q iff ϕ is satisfiable. W.l.o.g., we can assume that E contains one of $\{x, \overline{x}\}$ for each $x \in X$ (E simulates a total truth value assignment; admissible sets containing q might not include for each variable a corresponding argument, but then we can extend such sets). It holds that $\hat{E}$ is not spurious iff there is an $E' \in \mathit{adm}(E)$ with $m(E') = \hat{E}$ iff E' contains q. □

For showing Proposition 6, we make use of the following result.

Proposition 1. *Deciding whether some $\hat{E} \in \hat{adm}(\hat{F})$ in a given clustered AF $\hat{F}$ with some $a \in \hat{E}$ is spurious in showing the credulous acceptance of a under admissibility w.r.t. a given AF F is Σ_2^P-complete.*

Proof. For membership in Σ_2^P, some $\hat{E}$ containing a can be guessed and checked by Proposition 5 with a coNP oracle in polynomial time. The Σ_P^2-hardness is shown by a reduction from evaluating a QBF $\phi = \exists X \forall Y E(X, Y)$ where $E(X, Y) = \bigvee_{i=1}^{k} c_i$ is a DNF of conjunctions $c_i = l_{i_1} \wedge \cdots \wedge l_{i_{n_i}}$ over vocabulary X and Y where without loss of generality in each c_i some atom from Y occurs. Let $F = (A, R)$ be given by $A = \{x, \overline{x}, x' \mid x \in X\} \cup \{y, \overline{y}, y' \mid y \in Y\} \cup \{c \in C\} \cup \{q\}$ and $R = \{(x, \overline{x})(\overline{x}, x)(x, x')(\overline{x}, x') \mid x \in X\} \cup \{(y, \overline{y})(\overline{y}, y)(y, y')(\overline{y}, y') \mid y \in Y\} \cup \{(\overline{z}, c) \mid z \in X \cup \overline{X} \cup Y \cup \overline{Y}, z \in c, c \in C\} \cup \{(z', q) \mid z \in X \cup Y\} \cup \{(c, q) \mid c \in C\}$. Further, let $m(y) = m(\overline{y}) = \hat{y}$ for each $y \in Y$, and the rest of the arguments are mapped to themselves. We claim that there exists a $\hat{E} \in \hat{adm}(\hat{F})$ containing q which is spurious iff ϕ is satisfiable. Due to the construction of $\hat{F}$, $\hat{E}$ should contain x or $\overline{x}$ for each $x \in X$ and also the cluster $\hat{y} = \{y, \overline{y}\}$. When we try to build some $E \in \mathit{adm}(F)$ to match $\hat{E}$, the existence of x or $\overline{x}$ and picking an assignment on $y, \overline{y}$ to match the cluster $\hat{y}$ will definitely defend q from the x' and y' for each $x \in X, y \in Y$. Thus the spuriousness would have to occur due to some c not being attacked (and thus q not being defended) whenever an assignment is picked for $y, \overline{y}$. This means that such a c is not being attacked by x or $\overline{x}$. Thus, we can conclude that in fact ϕ is satisfiable, since for the given assignment on X from $\hat{E}$ we can pick any assignment on Y so that some conjunction is satisfied. The reverse is also easily seen, as we can construct some $\hat{E}$ according to the assignment on X which satisfies ϕ, which becomes spurious. □

Proof of Proposition 6. This result comes as a corollary of Proposition 1. □

Proof of Proposition 8. Assume the conditions of the proposition hold. Then $m(\sigma(F)) = \hat{\sigma}(\hat{F}) = m(\sigma(F'))$ (first item). Let $E \in \sigma(F)$. Then there is an $E' \in \sigma(F')$ with $m(E) = m(E')$. Recall that we assumed that singletons map to themselves. This means that $E \cap S \subseteq m(E) = m(E')$, and also that $E \cap S = m(E) \cap S$ (all singletons occurring in $m(E)$ must be part of E). The same holds for E': $E' \cap S = m(E')$. Then $E \cap S = m(E) \cap S = m(E') \cap S = E' \cap S$. The claim of the proposition (second item) follows. □

Proof of Proposition 9. We have $a \in A$ is credulously accepted in F under σ iff there is an $E \in \sigma(F)$ with $a \in E$ iff there is an $\hat{E} \in \hat{\sigma}(\hat{F})$ with $a \in \hat{E}$ iff a is credulously accepted in $\hat{F}$ under $\hat{\sigma}$. □

The proofs of Proposition 10, Proposition 11, and Proposition 12 follow from the definition of semantics being abstracting.